Standing Wave $Q \to \infty$

In this case the magnitude and phase of u_1 at the outer edge of the sublayer match the velocity at the outer edge of the boundary layer. Hence, the velocity fluctuations in the external stream are transmitted without change down to the outer edge of the sublayer. This is the (trivial) outer inviscid solution for this special case. The phase angle at the wall is +45 deg relative to the phase of the external velocity.

Wave Traveling with Finite Speed $Q > \bar{U}$

The magnitude of u_1 at the outer edge of the sublayer does not now match that at the outer edge of the boundary layer, although the phase does. The unsteady component of flow in the boundary layer outside the sublayer is governed at high frequencies by a first-order inviscid ordinary differential equation whose (purely real) coefficients contain functions derived from the Blasius solution for the mean flow.[4] This is the first-order outer equation in a matched asymptotic expansion. Numerical solutions for a range of values of $Q/\bar{U}$ are given in Ref. 4. There is again no change of phase through this outer region, and the value of $\bar{u}_1$ at its inner edge is equal to $(1 - \bar{U}/Q)$, asymptotically matching the sublayer solution. In the special case when $Q = \bar{U}$, the opposing contributions to the unsteady pressure gradient exactly cancel out, and the freestream oscillation is transmitted inwards through the boundary layer entirely by viscous diffusion. The high-frequency approximation is inapplicable.

Wave Traveling with Speed $Q < \bar{U}$

In this case neither the magnitude nor the phase of u_1 at the outer edge of the sublayer matches those quantities at the outer edge of the boundary layer. At the sublayer edge, u_1 is 180-deg out of phase with U_w, and the phase at the wall is −135 deg. The same inviscid equation as before applies outside the sublayer, but there is now a logarithmic singularity at the point in the outer layer where the velocity in the mean boundary layer is equal to Q. The phase angle is constant at −180 deg between the edge of the sublayer and this point and then gradually increases from −180 deg to zero between the singularity and the outer edge of the boundary layer. In real flow this singularity is smoothed out in a second viscous layer, termed the critical layer, an analysis of which is outlined in Ref. 4.

In conclusion it is observed that Lin's procedure for calculating the modification to the mean flow caused by the fluctuations, which was invoked by Greenblatt and Damelin, relies on the fact that for standing waves the sublayer solution and its continuation through the rest of the boundary layer do not depend on the mean flow. For traveling waves this independence does not exist, and therefore Lin's procedure would only be useful for small amplitudes at best.

Numerical solutions for high frequency confirm the structural features described earlier.[4,5]

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Comment on "Laminar Flow Past Three Closely Spaced Monodisperse Spheres or Nonevaporating Drops"

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Nomenclature

A	= constant in Table 1
C_{Ds}	= total drag coefficient of a single sphere
$C_{D,k}$	= total drag coefficient of sphere k
d	= distance between two spheres, as ratio of sphere diameter
d_{12}	= distance between spheres 1 and 2, as ratio of sphere diameter
d_{23}	= distance between spheres 2 and 3, as ratio of sphere diameter
Re	= Reynolds number
α_k	= ratio of effective drag coefficient of sphere k to that of isolated sphere

I. Introduction

A PAPER by Ramachandran et al.[1] presents a computational study of the way in which fluid flow deviates from that past an isolated sphere when two other identical spheres are present along the axis of flow. They conclude that the drag coefficient depends upon the Reynolds number and the spacing of the spheres, this spacing being the dimensionless ratio of the intersphere distance to the sphere diameter. The extent of the drag reduction was calculated for intersphere distances from 2 to 6 sphere diameters, and for Reynolds numbers between 1 and 200. The authors used curve-fitting techniques to derive the following equations, which have been adopted by a number of other workers:

$$\alpha_1 = \frac{C_{D,1}}{C_{Ds}} = 1 - 0.096 Re^{0.2475} d_{12}^{-0.965} \exp\left(\frac{0.4764}{d_{23}}\right) \tag{1}$$

$$\alpha_2 = \frac{C_{D,2}}{C_{Ds}} = 1 - Re^{0.1593}\left(\frac{0.2932}{d_{12}^{0.4876}} + \frac{0.1341}{d_{23}^{0.4242}}\right) \tag{2}$$

$$\alpha_3 = \frac{C_{D,3}}{C_{Ds}} = 1 - 0.325(\ln Re + 1)^{0.603} \exp\left(\frac{-0.282}{d_{12}}\right) d_{23}^{-0.385} \tag{3}$$

They reported[1] that "The average curve-fitting errors, i.e., the deviations of $C_{D,k} \cdots$ from the computer generated values, range from 2.4 ($k = 1$) to 4.7% ($k = 3$). Maximum errors are always less than 9% and may occur at low Reynolds numbers ($Re < 20$)." This may be considered a good fit, and suggests that the modeling will be of use in important applications such as the study of dusts and sprays. On the other hand it does not justify the reporting of constants to four significant figures.

II. Sensitivity of the Equations

If we take values of $Re = 50$ and $d_{12} = d_{23} = 4$, it can be seen that the constants in Eqs. (1–3) do not greatly affect the overall result by being rounded. For example, $Re^{0.2475} = 2.633$ and $Re^{0.25} = 2.659$; $d^{-0.946} = 0.269$ and $d^{-1} = 0.250$; $\exp(0.4764/d) = 1.126$ and $\exp(0.5/d) = 1.133$. These are differences of 1, 7.6, and 0.6%.

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Table 1 Values of exp(A/d)

A	d = 2	d = 4	d = 6
0.50	1.284	1.133	1.087
0.4764	1.269	1.126	1.083
−0.282	0.868	0.932	0.954
−0.30	0.861	0.928	0.951
−0.50	0.779	0.882	0.920

It is clear that, because the index on the Reynolds number is small, this portion of the equation is not very sensitive either to the exact value of the index or the Reynolds number. Thus, in Eq. (1) for Re from 1 to 200, the value varies from 1 to 3.71; in Eq. (2), it varies from 1 to 2.33; and in Eq. (3), from 0.80 to 2.73. The exponential function is not particularly sensitive to the exact value of the constant given in Eqs. (1–3) for intersphere distances of 2–6, as shown in Table 1. Thus in Eq. (1) the value of the exponential factor is a little above unity, whereas in Eq. (3) it is a little below unity.

III. Proposed New Form of the Equations

By using a spreadsheet to calculate values of Eqs. (1–3) for $2 < Re < 200$ and $2 < d_{12}, d_{23} < 6$, I found that a close match could be provided by the following equations, which give a better sense of the relationships involved:

$$\alpha_1 = \frac{C_{D,1}}{C_{Ds}} = 1 - 0.1\frac{Re^{\frac{1}{4}}}{d_{12}}\exp\left(\frac{0.5}{d_{23}}\right) \tag{4}$$

$$\alpha_2 = \frac{C_{D,2}}{C_{Ds}} = 1 - 0.14Re^{\frac{1}{6}}\left(\frac{2}{\sqrt{d_{12}}} + \frac{1}{\sqrt{d_{23}}}\right) \tag{5}$$

$$\alpha_3 = \frac{C_{D,3}}{C_{Ds}} = 1 - 0.5\frac{Re^{\frac{1}{6}}}{\sqrt{d_{23}}}\exp\left(\frac{-0.5}{d_{12}}\right) \tag{6}$$

Values calculated from Eqs. (4) and (5) differed from those calculated from Eqs. (1) and (2) by, at most, 0.3 and 3%, respectively. Equation (6) gave results within 10% of Eq. (3) up to $Re = 100$, rather more for extreme combinations of d_{12} and d_{23} at $Re = 200$. For example, at $Re = 50$, $d_{12} = d_{23} = 4$; then, values from Eqs. (1), (2), and (3) are 0.925, 0.586, and 0.536, respectively; from Eqs. (4), (5), and (6), values are 0.925, 0.597, and 0.577, respectively. The above have been compared only with the curve-fitting equations of Ramachandran et al., not with their data. The constants proposed in this communication are to one or two significant figures. A better fit might be obtained by comparison with the original data, and a precision of two significant figures thereby justified.

IV. Interpretation of New Equations

The form of the equations may be related to the theory of wakes produced by Moore[2] for bubbles (although the slip conditions are different from rigid spheres). According to Moore, there is a region to the rear of a bubble in which the vorticity from the boundary layer is transferred to the wake. This has a linear size dependent upon $Re^{-1/6}$ and velocity difference from the potential flow dependent upon $Re^{-1/3}$. Moore comments that these powers may seem surprising, but seem to be attributable to the three-dimensional nature of the flow. No such powers would arise for flow past a two-dimensional bubble. A term $f(Re^{1/4}/d)$ is derived by Harper[3] from Moore's theory for drag resulting from the wake between two or more bubbles.

Equations (4) and (6) both show the end sphere experiencing drag reduction (Re and d factor) attributable to the adjacent sphere, which is modified by the presence of a third sphere (exponential factor varies around unity). Equation (5) shows that the center sphere is affected by both of the others, the effect of the leading sphere being twice that of the trailing sphere for the same intersphere distances.

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Reply to M. J. Pitt

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THE simplified drag equations for laminar flow past three closely spaced monodisperse spheres proposed by Pitt are appreciated because of their ease of use and formal interpretation for bubbly flows. Perhaps a limit of $Re \leq 100$ should be suggested because of possible nonlinear error accumulation at higher Reynolds numbers and large particle spacings.

Drag and Nusselt number correlations extended to spherical and nonspherical droplets with mass transfer (i.e., blowing) may be found in Refs. 1–4.

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